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The Approximate Solution of 2D Dirichlet Problem in Doubly Connected Domains

Kazan Federal University, 420008, Kremlevskaya 18, Kazan, Russia

Abstract

© 2018 Atallah El-shenawy and Elena A. Shirokova. We propose a new method for constructing an approximate solution of the two-dimensional Laplace equation in an arbitrary doubly connected domain with smooth boundaries for Dirichlet boundary conditions. Using the fact that the solution of the Dirichlet problem in a doubly connected domain is represented as the sum of a solution of the Schwarz problem and a logarithmic function, we reduce the solution of the Schwartz problem to the Fredholm integral equation with respect to the boundary value of the conjugate harmonic function. The solution of the integral equation in its turn is reduced to solving a linear system with respect to the Fourier coefficients of the truncated expansion of the boundary value of the conjugate harmonic function. The unknown coefficient of the logarithmic component of the solution of the Dirichlet problem is determined from the following fact. The Cauchy integral over the boundary of the domain with a density that is the boundary value of the analytical in this domain function vanishes at points outside the domain. The resulting solution of the Dirichlet problem is the sum of the real part of the Cauchy integral in the given domain and the logarithmic function. In order to avoid singularities of the Cauchy integral at points near the boundary, the solution at these points is replaced by a linear function. The resulting numerical solution is continuous in the domain up to the boundaries. Three examples of the solution of the Dirichlet problem are given: one example demonstrates the solution with constant boundary conditions in the domain with a complicated boundary; the other examples provide a comparison of the approximate solution with the known exact solution in a noncircular domain.

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References

[1] I. Fredholm, "Sur une classe d'équations fonctionnelles," Acta Mathematica, vol. 27, no. 1, pp. 365-390, 1903.

[2] J. L.Hess and A.M.O. Smith, "Calculation of non-lifting potential flow about arbitrary three-dimensional bodies," Journal of Ship Research, vol. 8, no. 2, pp. 22-44, 1964.

[3] M. A. Jaswon, "Integral equation methods in potential theory," Proceedings of the Royal Society of London. Series A, vol. 275, pp. 23-32, 1963.

[4] J. L. Blue, "Boundary integral solutions of Laplace's equation," Bell Labs Technical Journal, vol. 57, no. 8, pp. 2797-2822, 1978.

[5] C. K. Hsieh and A. J. Kassab, "Complex variable boundary element methods for the solution of potential problems in simply and multiply connected domains," Computer Methods Applied Mechanics and Engineering, vol. 86, no. 2, pp. 189-213, 1991.

[6] C.-S. Liu, "A highly accurate collocation Trefftz method for solving the Laplace equation in the doubly connected domains," NumericalMethods for PartialDifferential Equations, vol. 24, no. 1, pp. 179-192, 2008.

[7] F. D. Gakhov, Boundary Value Problems, Oxford: Pergamon Press, 1966.

[8] P. N. Ivanshin and E. A. Shirokova, "Approximate conformal mappings and elasticity theory," Journal of Complex Analysis, Art. ID 4367205, 8 pages, 2016.

[9] N. I. Muskhelishvili, Singular integral equations, Springer Netherlands, Netherlands, 1958.

[10] D. F. Abzalilov and E. A. Shirokova, "The approximate conformal mapping onto simply and doubly connected domains," Complex Variables and Elliptic Equations. An International Journal, vol. 62, no. 4, pp. 554-565, 2017.

[11] E. A. Shirokova, "Onthe approximate conformalmapping of the unit disk on a simply connected domain," Russian Mathematics (Iz. VUZ), vol. 58, no. 3, pp. 47-56, 2014.

[12] F. G. Tricomi, Integral Equations, Interscience, New York, NY, USA, 1957.

[13] R. G. Cooke, Infinite Matrices &sequence spaces, Dover publications Inc, New York, NY, USA, 2014.

[14] Z.-Q. Wang, S. Li, Y. Ping, J. Jiang, and T.-F. Ma, "A highly accurate regular domain collocation method for solving potential problems in the irregular doubly connected domains," Mathematical Problems in Engineering, Art. ID 397327, 9 pages, 2014.